Assignment 3

This homework is due Friday Feb 13.

There are total 50 points in this assignment. 45 points is considered 100%. If you go over 45 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 1.6–2.1 of Textbook.

(1) [15pt] Sketch the sets below. In each case state, with reasons, which of the following terms apply to these sets: open; connected; domain; region; closed set; bounded.
 (a) $\{z : \mathrm{Re}(z) > 1\}$.
 (b) $\{z : -2 \le \mathrm{Im}(z) < 3\}$.
 (c) $\{z : |z - 1 + i| \le 2\}$.
 (d) $\{z : |z + 3i| > 1\}$.
 (e) $\{re^{i\theta} : 0 < r < 1 \text{ and } -\frac{\pi}{2} < \theta < \frac{\pi}{2}\}$.
 (f) $\{re^{i\theta} : r > 1 \text{ and } \frac{\pi}{4} < \theta < \frac{\pi}{3}\}$.
 (g) $\{z : |z| < 1 \text{ or } |z - 3| < 1\}$.

(2) [5pt] Find a parametrization of a curve that is a straight line (a) from the point $-i$ to the point $1 + i$, (b) from the point 2 to the point $1 + i$.

(3) [5pt] Sketch the curve $z(t) = t^2 + 2t + i(t + 1)$ (a) for $-1 \le t \le 0$, (b) for $1 \le t \le 2$. (*Hint:* Use $x = t^2 + 2t$, $y = t + 1$ to eliminate t, therefore getting a condition on x, y.)

(4) [5pt] Find a parametrization of the curve that is a portion of the circle $|z - 1| = 1$ that joins $1 - i$ to $1 + i$ if (a) the curve is the right semicircle, (b) the curve is the left semicircle.

(5) [5pt] Find $f(-1 + i)$ for the following functions
 (a) $f(z) = z + z^{-2} + 5$.
 (b) $f(z) = \frac{1}{z^2+1}$.
 (c) $f(z) = f(x + iy) = x + y + i(x^3y - y^2)$.
 (d) $f(z) = z^2 - 4z\bar{z} - 5\,\mathrm{Re}(z) + \mathrm{Im}(z)$.
 (e) $f(z) = z^{19} - 4z^8 + 9z^3$ (in this item, use polar coordinates).

(6) [5pt] Express the following functions in the form $u(x, y) + iv(x, y)$.
 (a) $f(z) = z^3$.
 (b) $f(z) = \bar{z}^2 + (1 - 4i)z$.
 Express the following functions in the polar coordinate form $u(r, \theta)+iv(r, \theta)$.
 (c) $z^5 + \bar{z}^5$,
 (d) $z^3 + z^{-3}$.

(7) [5pt] Let $f(z) = f(x + iy) = e^x \cos y + ie^x \sin y$. Find
 (a) $f(0)$, (b) $f(i\pi)$, (c) $f(i\frac{2\pi}{3})$, (d) $f(2 + i\pi)$, (e) $f(3\pi i)$.
 Is f a one-to-one function? Explain your answer.

(8) [5pt] For $z \neq 0$, put $f(z) = \ln r + i\theta$, where $r = |z|$, and $\theta = \mathrm{Arg}\, z$. Find
 (a) $f(1)$, (b) $f(-2)$, (c) $f(1 + i)$, (d) $f(-\sqrt{3} + i)$.
 Is f a one-to-one function? Explain your answer.